Enhancing Morris' method

F. Campolongo, J. Cariboni, and A. Saltelli
European Commission, Joint Research Centre, 21020 Ispra, Italy

A sensitivity method widely used to screen factors in models of large dimensionality is the design proposed by Morris (1991). The Morris method deals efficiently with models containing hundreds of input factors without relying on strict assumptions about the model (such as for instance additivity or monotonicity of the model output). The method is relatively economical in terms of model evaluations required as with Morris the number of model executions is a linear function of the number of input factors involved.
The experimental plan proposed by Morris is composed of individually randomized 'one-factor-at-a-time' experiments: the impact of changing one factor at a time is evaluated in turn. Each input factor may assume a discrete number of values, called levels, which are chosen within the factor range of variation.
The sensitivity measures proposed in Morris' original work of 1991are based on what is called an elementary effect. The elementary effect for the ith input is defined as follows. Let Δ be a predetermined multiple of 1/(p-1). For a given value of x, the elementary effect of the ith input factor is defined as

$$d_i(\mathbf{x}) = \frac{[y(x_1,..,x_{i-1},x_i+\Delta,x_{i+1},..,x_k)-y(\mathbf{x})]}{\Delta}$$

where $\mathbf{x}=(x_1,x_2,...,x_k)$ is any selected value in Ω such that the transformed point (x + $\mathbf{e}_i$ Δ), where $\mathbf{e}_i$ is a vector of zeros but with a unit as its ith component, is still in Ω for each index i=1,..,k.
The finite distribution of elementary effects associated with the ith input factor, is obtained by randomly sampling different x from Ω, and is denoted by F_i.
In Morris (1991), two sensitivity measures were proposed for each factor: μ, an estimate of the mean of the distribution F_i, and σ, an estimate of the standard deviation of F_i. A high value of μ indicates an input factor with an important overall influence on the output. A high value of σ indicates a factor involved in interaction with other factors or whose effect is non-linear.
Here we propose to consider a third sensitivity measure: μ*, which is an estimate of the mean of the distribution of the absolute values of the elementary effects (here denoted as G_i).
In our view μ* is the most appropriate to rank factors in order of importance. The reason is that if the distribution F_i contains negative elements, which occurs when the model is non-monotonic, when computing its mean some effects may cancel each other out. Thus a factor which is important but whose effect on the output has an oscillating sign may be erroneously considered as negligible.
The Morris method has several advantages: it is simple, easy to implement, and results are easily interpreted. Furthermore it is economic in the sense that it requires a number of model evaluations that is linear in the number of model factors. Apparently it could be regarded as a local method because it relies on a sensitivity measure (the elementary effect), which uses incremental ratios and is therefore a local measure. However, the final measure μ* is obtained by averaging the absolute values of several elementary effects computed at different points of the input space. In this sense, as it attempts to explore several regions of the input space, the method can be regarded as global.
Next, we attempted to make a comparison between the sensitivity measures μ* and σ, and the class of the so-called variance based sensitivity measures, also known as importance measures or sensitivity indices.

Variance based measures choose as a measure of the main effect of a factor X_i on the output, an estimation of quantity $\frac{V\left(E\left(Y\middle|X_i = x_i^*\right)\right)}{V(Y)}$. Reasons for this choice are explained as it follows. If the aim is to rank factors according to the amount of output variance that is removed when we learn the true value of a given input factor X_i, the factors could be ranked according to $V\left(Y\middle|X_i = x_i^*\right)$, the variance obtained by fixing X_i to its true value x_i^*. This variance is taken over all factors but factor X_i. The problem is that we do not know what x_i^* is for each X_i and therefore it is reasonable to take the average of the above measure over all possible values x_i^* of X_i, i.e., over $E(V(Y|X_i))$. Given that $V(Y)$ is a constant and $V(Y) = V\left(E\left(Y\middle|X_i\right)\right) + E\left(V\left(Y\middle|X_i\right)\right)$, betting on the lowest $E(V(Y|X_i))$ is equivalent to betting on the highest $V(E(Y|X_i))$. The sensitivity measure above is thus obtained by normalising this quantity by the output unconditional variance $V(Y)$ to obtain $\frac{V\left(E\left(Y\middle|X_i = X_i^*\right)\right)}{V(Y)}$, which is known in the literature as the "first order effect" of X_i on Y, denoted by S_i.

Another sensitivity measured based on the variance decomposition is the total sensitivity index, S_{T_i}. The total index is defined as the sum of all effects involving the factor X_i. S_{T_i} is estimated by the quantity $E(V(Y|X_{-i}))$, where the symbol *-i* denotes all but index *i*.

Variance based techniques have several desirable properties. They are "model free", in the sense of independent from assumptions about the model such as linearity, additivity and so on. They are global, i.e. they explore the entire interval of definition of each factor and the effect of each factor is taken as an average over the possible values of the other factors. They are usually quantitative, which is they can tell how much factor *a* is more important than factor *b*. They are able to treat grouped factors as if they were single factors, a property of synthesis that may be essential for the agility of the interpretation of the results. The choice to make use of a screening method such as Morris rather than of a variance-based method is due merely to computational reasons, i.e. in cases where the cost of computing a variance-based measure is unaffordable. The Morris method is much cheaper, in terms of model evaluations, than the variance based measures, and is therefore suitable for models that are computationally expensive or that contain a large number of input factors.

In this work we compared the variance based measures and the sensitivity measures μ^* and σ. Both theoretical reasoning and experimental results showed that the measure μ^* is the best parallel of the total sensitivity index S_{T_i} while σ is the parallel of the measure ($S_{T_i} - S_i$).

Thirdly, we tried to extend to the Morris measure a desirable property of the variance–based methods: the capability to treat group of factors as if they were single factors. The Morris method was extended to perform also when model input factors are divided into groups with good results.

We conclude by saying that the sensitivity measures μ^* (a modified version of the Morris μ) and σ (proposed by Morris) can be effective in the same settings where the variance based are. Their use is recommended when the problem is such that the computation of variance-based measures is not affordable. Although less accurate, they can be interpreted similarly to the S_{T_i} and to the ($S_{T_i} - S_i$). Furthermore they have several of the desirable properties of a variance-based measure, including the capability to work with group of factors.